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You've Got Mail! – Written Communication and Feedback in Mathematics

Ana Barbosa 

Instituto Politécnico de Viana do Castelo, Portugal

Isabel Vale 

Instituto Politécnico de Viana do Castelo, Portugal

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Ana Barbosa, Isabel Vale

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Abstract

This paper describes a study that aims to understand and characterize the written communication of future teachers through a pen pal experience with elementary education students, in particular the nature of their feedback. To carry out this investigation we followed a qualitative methodology and collected data through observation, interviews and written productions. The participants were seven pre-service teachers that attended a Master's Degree Course in Primary Education (6-12 years old) who interacted through letter correspondence with 3rd grade students. Results show that the pre-service teachers valued this experience, considering it useful and effective in the development of written communication. They also had the opportunity to identify the importance of more general aspects, such as the adequacy of the discourse, the need to acknowledge the curricular guidelines and the features of the educational context. The type of feedback given in the written commentaries was diversified, trying to meet the main characteristics of evaluative writing, being intentional, personalized and identifying aspects to improve through self-regulation.

Introduction

Pen pal experiences are useful in the development of language skills and in establishing rich and meaningful interactions through letter exchange. The application of this strategy in the field of mathematics education, creates the opportunity to enhance and develop written communication. In the mathematics classroom, this dimension of communication is not as privileged as verbal communication, which expectedly emerges more often, associated to the use of natural language. In addition, there is frequently some resistance from students in writing short texts about mathematical ideas, claiming that they do not understand the purpose of this type of communication when oral speech is easier and faster (e.g. Phillips & Crespo, 1996; Thompson & Chappell, 2007). Writing in the mathematics classroom is an activity where the majority of younger students have little or no experience (e.g. Haltiwanger & Simpson, 2013; Phillips & Crespo, 1996). Letter correspondence, among other strategies, can help bridge this gap by facilitating students' engagement in the writing process by having a genuine reason to communicate in this way (e.g. Crespo, 2003; Norton & Katsberg, 2012; Phillips & Crespo, 1996). On the other hand, being an individual and personalized experience, the teacher has the opportunity to communicate with all students and access their perspectives and knowledge in mathematics, which is more difficult to achieve though discussion or other verbal interactions.

In addition to promoting written communication, this type of strategy also allows the teacher to engage in

evaluative writing, often called feedback, in its written form. Teachers' feedback is critical for students' learning and self-regulation. It is a fundamental tool of formative assessment (Havnes et al., 2012) in the sense that provides information with the intention to alter students' thinking or behavior to improve learning. However, it is found that teachers rely primarily on oral feedback, a context that may be somewhat reductive in identifying more individualized difficulties and positive aspects in the productions of students. From the students' point of view, the comments written by the teacher can help them identify strengths and weaknesses of their written productions and, consequently, progress in learning while reviewing what they have done (Havnes et al., 2012; Katsberg, Lischka, & Hillman, 2020).

As previously discussed, the role of the teacher is preponderant in valuing written communication, but also in diversifying the types of feedback given to students. This implies the use of appropriate teaching methodologies to highlight written communication and feedback. Teacher training is a privileged context to discuss the importance of these aspects in the teaching and learning of mathematics, and it is fundamental that future teachers can benefit from such experiences in their education. Thus, the aim of this study is to understand and characterize the written communication of pre-service teachers throughout a pen pal experience with primary school students, in particular the nature of their feedback. To better address the stated problem, we particularly seek to answer the following research question: What traits of written mathematical feedback are evidenced by future elementary school teachers in a pen pal experience? What potentialities and difficulties are identified in the formulation of mathematical feedback through a pen pal exchange?

Communication in the Mathematics Classroom

Communication is a fundamental component of mathematics teaching and learning. This idea is embodied in a wide variety of curricular documents, both national and international. It is a transversal ability to all mathematical activity that contributes to the construction of meanings, the consolidation and dissemination of ideas. By fostering communication in the mathematics classroom, students have the opportunity to reflect, clarify and expand their knowledge of mathematical relationships (OME, 2005; NCTM, 2000, 2014). In recent years, the focus of learning has been set on students as creators of information and not so much on students as consumers of that information. This paradigm shift is largely due to the need to develop skills recognized as essential in the 21st century, in which we identify communication. This goal can only be achieved if the teacher promotes a classroom culture that allows students to be at the center of learning, privileging interactions and collaboration, aspects intrinsically linked to communication.

Analyzing the most recent curricular documents in the scope of mathematics education, it is possible to perceive the determinant role of communication, in its different strands. A document recently published by the Ministry of Education in Portugal (DGE, 2018), states that the development of mathematical communication is an essential learning ability, highlighting the importance of students acquiring the proper vocabulary and language of mathematics and developing the ability to communicate in mathematics in order to be able to describe, explain and justify, orally and in writing, their ideas, reasoning, as well as results and conclusions. The document *Principles to Actions: Ensuring Mathematical Success for All*, published by the National Council of

Teachers of Mathematics (NCTM, 2014), defines eight teaching practices, which are considered fundamental for the development of students' mathematical proficiency, some of which are directly linked to mathematical communication, namely: favoring a significant mathematical discourse; asking relevant questions; and using and relating mathematical representations.

Since Mathematics itself is considered a language that allows us to understand and represent the world, we can assume that there is a direct relationship between mathematical thinking and mathematical communication. As with any language acquisition, students learn mathematics and learn to communicate mathematically by speaking, listening, reading and writing (Thompson & Chappell, 2007). Given the different forms of mathematical communication, verbal communication is perhaps the most natural way for students to express emerging ideas. However, written communication is also of special importance, as it provides students with a record of their own thinking, enabling reflection about the work produced (NCTM, 2014). On the other hand, it gives the teacher insights into the students' thinking, allowing the analysis of the thinking processes exposed on paper, as well as the identification of possible misconceptions. Overall, writing can be seen as a process that enhances the development of communication skills and mathematics proficiency. We can say that these forms of communication play a specific role in the mathematics classroom and, for this reason, should be widely explored. On the one hand, verbal communication is the most used in the interaction between students and the teacher and in moments of collective discussion. As for written communication it is enabled when students are required to make written records related to a given task or to elaborate short texts about certain mathematical contents (Haltiwanger & Simpson, 2013). Because the focus of this study stands on written communication, we consider pertinent to emphasize some aspects that are inevitably inherent. Writing in the mathematics classroom can help students deepen mathematical exploration processes and reflect on their own mathematical processes. It also helps them to organize their knowledge and understand the questions posed at a deeper level (Braun, 2014). It can enhance the learning of specific vocabulary and its proper use (NCTM, 2014). When students communicate their thinking in writing, they tend to clarify their ideas and provide valuable information to the teacher. Research also shows that writing supports metacognition (e.g. Pugalee, 2015), as it provides a level of reflection that promotes a particular attention to one's thinking about the mathematical processes used. This awareness creates and opportunity for self-regulation and plays an important role in the selection of appropriate information and/or strategies while solving mathematical problems.

It is clear that writing as an activity in the mathematics classroom, can enhance learning and promote the development of important abilities, but we also have to be aware that it can be used for different purposes. Not all types of writing have the same potential to leverage students' learning and particularly mathematical reasoning, presenting different characteristics. Mathematical writing may be of different types having a diversity of purposes, namely it can be: exploratory, when the goal is to make sense of a problem, situation or one's own ideas; informative/explanatory, if the student wants to describe or explain mathematical ideas; argumentative, when the construction/critique of an argument is the main intent; creative, if the written production reports original ideas, problems and/or solutions, conveying fluency and flexibility in thinking (Casa et al., 2016). These different types of writing need to be stimulated by the teachers using prompts that provoke students' engagement in mathematical reasoning.

Overall, communication skills are refined as students: practice, explaining their ideas in a logical and coherent way; develop an awareness of the audience to which their respective productions are destined; and assess their communication skills through the processes of review and refinement (Haltiwanger & Simpson, 2013).

Written Feedback through a Pen Pal Exchange

Writing in mathematics can be used as a means of assessing students learning, either through diagnostic, formative or summative assessment. It can be used for diagnostic assessment when, for example, students provide personal texts about their mathematical experiences (Braun, 2014). On the other hand, teachers can use it to adjust their instruction and provide feedback to students in the context of their own work as part of formative assessment (Black & William, 2009; Chong, 2018; Santos & Semana, 2015). It may also lead to a more globalizing judgment, culminating in a mark when summative assessment is put into practice. Following the scope of this study we are interested in the second strand, associated with the practice of evaluative writing or written feedback.

Being a nuclear aspect of this work, it is important to understand what is meant by feedback, in particular its written form. Globally Hattie and Timperley (2007) consider that the main purpose of feedback is to reduce the gap between learning objectives and student achievement. It is conveyed through information provided to students with the intention of modifying their thinking or behavior and improving learning. So, it can be seen as a formative assessment practice, used to guide students towards specific learning goals and hence support learning (Black & William, 2009; Chong, 2018). The key elements of feedback stand on students receiving information on where are they going, how are they doing and where to go next (Hattie & Timperley, 2007). When mathematical knowledge is involved, teachers must clearly inform students about the goals of a certain task, focusing on mathematical ideas, and make suggestions about possible pathways, highlighting aspects related to processes and strategies used. “Getting underneath students’ understanding, finding out what they really think, is the starting point of all feedback, from whichever direction, because only then can the feedback be appropriately constructed to provide advice” (Hattie & Clarke, 2018, p.4). Also, an effective feedback must be product-oriented (e.g. about specific mathematical contents) and process-oriented (e.g. about the adequacy of a particular strategy), pointing out strengths and weaknesses in students’ mathematical activity (Butler & Winne, 1995), presenting comments about the processes applied or needed to solve a task and how the solution can be improved.

Feedback is undoubtedly a communicative process, whether conveyed in oral or written form, which translates into discursive interventions and/or recorded comments (Terroso et al., 2019). The main intention, in both forms of feedback, is that students recognize its purpose, meaning and value, so that it can be effective. But oral and written feedback are significantly different processes. The first is immediate, provided in the moment, in response to students’ actions and leaves no written record. Written feedback is also organized in response to students’ work but provides a record that can be revisited whenever at any time (Katsberg et al., 2020). There is another noteworthy difference and that is written feedback provides teachers with a means to address students’

individual needs, while also giving them a record of the information that they can save and go back to as needed. In this sense it can be considered as tangible, individualized and private.

It is critical to create opportunities to include written communication and meaningful feedback practices. Thus, exchanging letters through a pen pal experience emerges as a strategy that offers participants a rich and genuine experience in the sense that there is an effective and intentional interaction with regard to communication (e.g. Crespo, 2003; Norton & Katsberg, 2012). It is a context that encourages reading, writing and discussion of ideas that tends to engage participants more effectively. The fact that it is not a form of immediate communication implies that there is greater attention in the clarification of ideas and, consequently, the elaboration of a higher quality written production. This strategy is a collaborative process and requires equal attention from both parties. According to McCaffery (2012), the exchange of correspondence through letters contributes to the development of literacy, communication processes and, overall, learning, among other aspects of a more affective nature. It is an experience that arouses sincere interest and greater involvement in the activity to be developed. Participants tend to show pleasure in contacting with the recipient of the letters while simultaneously demonstrating an increase in their knowledge without constant teacher reinforcement. The discussion created between those involved enhances the understanding of concepts and motivates critical thinking.

In the context of teacher education, in addition to the attention with clarity and the correctness of writing, this strategy makes it possible to raise awareness of the appropriateness of the speech to the recipient. Letter correspondence is an opportunity for prospective teachers to access the thinking of the students with whom they communicate and to send feedback through evaluative writing. The nature and quality of feedback, in this context or any other, is crucial to a successful learning, since it can either be a powerful moderator in achieving improvement or have unintended negative implications on achievement or attitudes towards learning (Hattie & Timperley, 2007; Katsberg et al., 2020). That is why it is important to understand how the intervention should be carried out. Giving feedback is not an easy task, being sometimes difficult to distinguish which type of feedback will be the most appropriate in order to have a positive effect on learning (Hattie & Timperley, 2007). So, we must take into consideration a number of different factors in order to meet the desired effects. In general terms, feedback includes telling (or writing) students what they have done well and what they need to do to improve, also reminding them what they were aiming to achieve (NCCA, 2015). In terms of written feedback, it can be focused on form (writing aspects, like correctness or clarity) or content. At the content level we may find tasks centered indicators, as stated by Hattie and Timperly (2007): information about task performance (how well a task is understood or solved); information about the processes used (processes/skills/strategies required to solve a task); self-regulatory information (regulation of action/self-monitoring, leading to the comparison and adjustment of students' work in relation to the required standards or intent); and information on the self-level (the learner as a person, not related to task performance; focused on personal evaluations and affect – usually positive – about the learner; commonly associated with praise). Santos and Semana (2015) add some characteristics that can be interpreted in a more general sense, namely: be appropriate for each student; be descriptive and incisive in a given task; refer and acknowledge student effort; be dialogic; be informative; give clues for future action; encourage the students to re-analyze their answers. The dialogic nature of feedback, particularly written feedback, can be considered a fundamental feature in student-teacher interaction (or peer

interaction), as it is known to engage students more meaningfully in the assessment process which can facilitate self-regulation (Mulliner & Tucker, 2017).

The experience of letter exchange induces teachers to adopt a relational attitude when they write in a dialogic way, simultaneously trying to help students achieve a better mathematical performance, but also showing care by their interests and their individual reasoning (e.g. Crespo, 2003; Norton & Katsberg, 2012). So, this approach to feedback encompasses the attention to the self as well as to the mathematical ideas involved in the written productions.

Methodology

This study follows an interpretative qualitative methodology (Erickson, 1996). The choice for an interpretative paradigm is sustained by the fact that the main goal is to understand how the participants regard and perceive a specific situation. The participants were seven students of a master's course in primary education, the totality of students in that class. These pre-service teachers attended a unit course in Didactics of Mathematics that acted as the context for this study. They addressed themes associated with curricular management, in particular the types of tasks used in mathematics education and requirements in their selection, and the assessment of learning, namely evaluative writing/written feedback.

The opportunity arose to propose a ten weeks pen pal experience between these future teachers and 3rd grade students, providing them the contact with real students while applying knowledge acquired in the unit course. Each of the participants was paired with two students at random, and four letters were written by each of the elements involved in the correspondence exchange. In the first letter, the focus was on presenting themselves and getting to know the recipients, acquiring knowledge about their personal characteristics, their relationship with mathematics, what they were learning, difficulties felt, among other aspects. The following letters, in addition to being more specific, continuing the initiated dialogue, included mathematical tasks for the students involved to solve, whose solutions would be subjected to feedback. Despite these guidelines, the content of the letters and particularly the tasks sent were of the sole responsibility of the participants. During the pen pal exchange, the pre-service teachers read and reviewed the received letters, organizing their responses in a manner that should include feedback on students' work, writing comments and sending new tasks and challenges. This work was supported throughout the unit course, once a week, with discussion sessions, where the participants could share doubts about the content of the letter and/or the feedback itself. As can be seen, through this study the participants were able to use formative assessment as an integrated part of their instruction but also as a learning tool in the contact established with the students.

Data was collected in a holistic, descriptive and interpretive manner and included participant observation (classes), written productions (letters) and an interview with each participant at the end of the experience. The researchers were the teachers of the unit course in question, a fact that facilitated the accomplishment of the participant observation, accompanying the work of the students in the writing of their letters and in the reading of the received letters. At the end of the semester, a semi-structured interview was conducted with each of the

pre-service teachers in order to access their perceptions about the experience. In the data analysis, we used descriptive and content analysis methods (Miles & Huberman, 1994), trying to find behavioral patterns related to the quality/traits of written communication/feedback, the perceptions of future teachers about the potentialities and the difficulties of this experience.

Results and Discussion

The dynamics of the pen pal exchange allowed the records to be prepared with great care and attention by the participants in this study. There was time to read the letters, reflect on the response, and post the correspondence in an appropriate time window. Having little or no experience in writing formal letters, the future teachers initially showed difficulties with the expected dissertation structure of such a document, but quickly overcame this problem, having realized the need to write texts in the form of a dialogue, to foster communication with each correspondent. They also showed concern in adapting the language to the recipients, real students that they never met, mainly thinking if their ideas were clear enough or intelligible. However, at the same time, this type of communication was highlighted as positive by the future teachers who, with this experience, had the opportunity to contact with primary school students. Another general aspect that emerged from this experience was related to the tasks proposed to the students. When posing/selecting these tasks, to include in the letters, the participants realized the importance of knowing the curricular guidelines for 3rd grade Mathematics, as well as the contents these students were addressing at the moment, in order to properly adapt the proposals.

The tasks posed by the pre-service teachers varied between exercises and problems. The problems were of two types, two or more step problems and process problems, with predominance of the latter, aiming to increase the cognitive level of demand. Being aware, through the letters received, of the contents that their correspondents were working on at the moment (e.g. times tables, statistics concepts, roman numbers, relative position of line segments), some of these future teachers chose to propose tasks related to these contents. They understood that this was an opportunity for students to “apply” or to “revise” what they were learning, considering that “it might be helpful for them” and also wanted to guarantee that the tasks posed would not be out of step with their knowledge. As an example, figure 1 illustrates one of these situations. The task proposed by the future teacher was based on the contents identified by the correspondent 3rd grade student (see Figure 1).

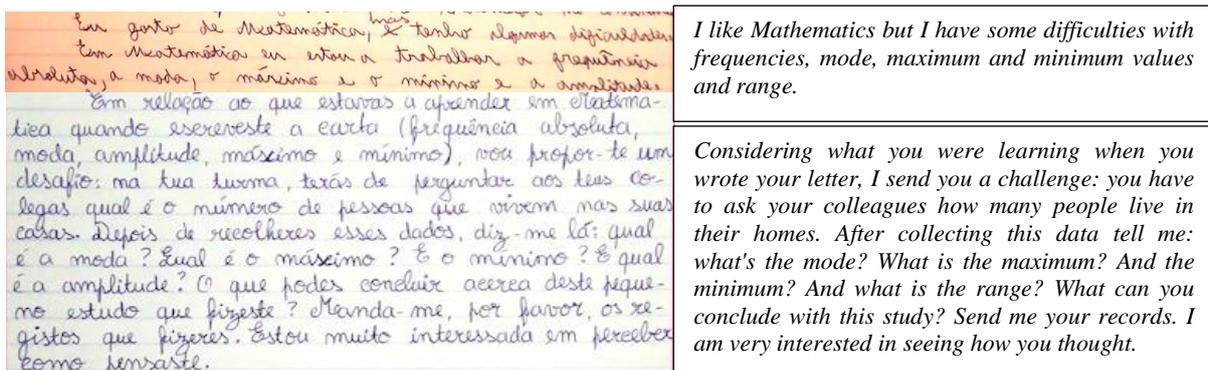


Figure 1. Example of a Task Emerging from the Contents

The predominance of process problems in the proposals sent by the future teachers can be justified by the fact that these tasks were considered as “more transversal”, in the sense that they do not necessarily have to “address a specific content”. Thus, the purpose in these cases was not so much the application of a content taught at the time of the experience, but the “development of mathematical reasoning”, valuing tasks with higher cognitive demand that could have “greater potential for written feedback”. We must emphasize that, in most cases, the context of these problems was related to personal aspects shared by the corresponding student (see Figure 2).

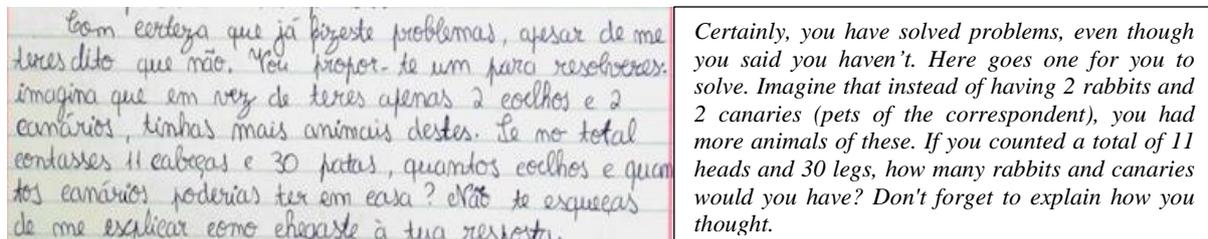


Figure 2. Example of a Process Problem

The type of feedback sent by the future teachers was quite diverse, that also showed concern with the adequacy of the written comments to each student, trying to personalize the content of the letters and continue the dialogue, initiated in the first letter, throughout the four interactions. Analyzing the letters, we were able to identify some specific traits of evaluative writing/feedback, which we will present and discuss. These pre-service teachers believed in the importance of praise to engage students, frequently referring and acknowledging students' efforts in the letters. So, they often highlighted positive aspects of the written productions. This is an important aspect that should be clear in teacher education, a teacher must not only emphasize what is wrong or should be improved, but also identify the strengths of the work presented by the students (orally or in writing). These pre-service teachers realized the relevance of including comments of this nature in the letters sent, situating the information at the self-level (see Figure 3).

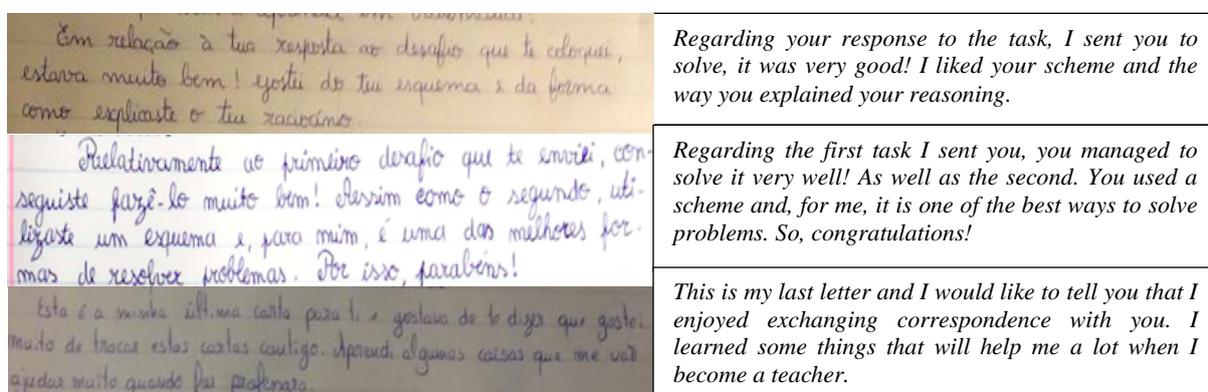


Figure 3. Excerpts from Letters Acknowledging Students' Efforts

It should be noted that, in some cases, the feedback was not informative to the corresponding student, and we can even say that it was rather superficial, expressed in comments like: “I really enjoyed your letter!”; “The answer to the first task is correct”; “You have an error in one of the operations”. Despite contemplating information about task performance and about processes used, feedback like this does not help students

understand what to adjust and exactly what they did right. However, most of the pre-service teachers chose to write clearer comments about what they positively stressed, showing improvement throughout this exchange: “You were able to solve the problem correctly. You used a scheme and explained your reasoning well.”; “You have used a good strategy to solve the problem. Congratulations! However, you forgot to present the answer. It is very important that you don’t skip this step when you are solving a problem”; “You used correct procedures to collect and interpret data, however you forgot to include yourself in the sample”. These examples, besides presenting specific information about task performance and processes used, also identify aspects in the scope of self-regulatory information, which is fundamental to promote learning.

In several situations, the comments written in the letters pointed out clues for future action, seeking self-regulation by the targeted student and ultimately to promote learning. Figure 4 shows an example illustrating this situation. Based on a student's doubt (6 times table), a comment was made with the intention of contributing to his mathematical knowledge (deducing a rule from the 3 times table), a comment that was valued by the recipient. The 6 times table was pointed out by the student as being difficult to learn, aspect that was not overlooked by his correspondent, leading him to formulate a suggestion that, in his opinion, could be useful to help the student reflect about that specific situation.

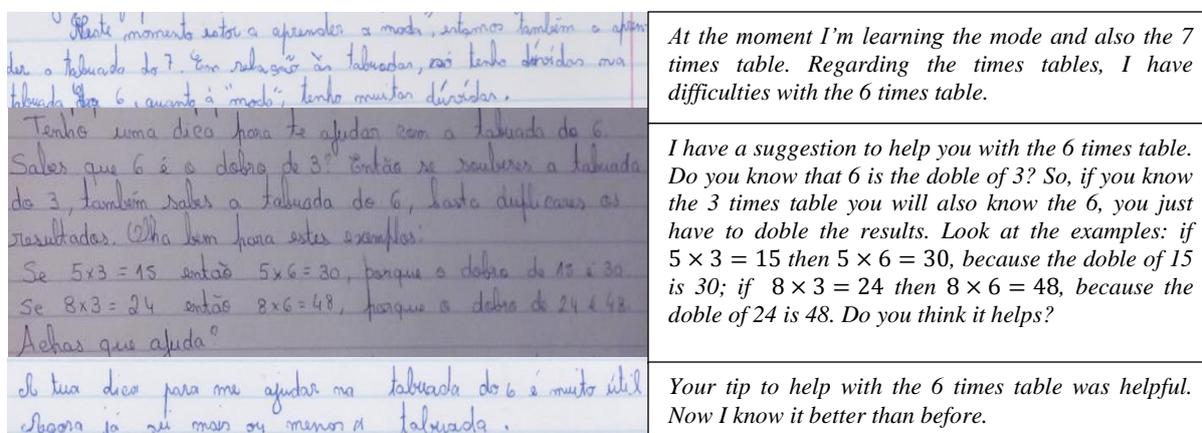


Figure 4. Excerpts from a Letter with Clues for Future Action

The pre-service teachers who participated in this study also showed, through the comments made, the importance given to the explanation of the reasoning in the written communication. Overall, in the tasks sent to the students, the need to request a justification was noticeable, since not every correspondent clarified their thinking. This situation was reflected in expressions/questions such as: “Explain how you thought”; “How do you know?”; “Explain how did you reach your answer”; “I am very interested in understanding how you thought”. These students were not used to this kind of records in the mathematics classroom, writing only the basic of their reasoning on paper, mainly calculations and some words. The lack of written communication habits results in the difficulty in clearly expressing the ideas to the reader. As a result, in many of the letters, the future teachers felt the need to encourage students to explain their reasoning further or to clarify their ideas, motivating them to reanalyze their answers.

Throughout the correspondence exchange, and as expected, errors were also identified in the students'

productions. These mistakes were carefully identified and the correspondents were informed about the inaccuracies committed, allowing them to overcome their difficulties through the comments made. In these cases, feedback was written informatively for the reader. The same thing happened in situations where the participants felt the need for clarification or greater mathematical correction. Also, we noticed that the feedback carried out by these future teachers had always a dialogical structure, like they were “talking” to their recipients trying to follow the structure of a letter. The fact that the correspondents did not know each other and that communication was established through letters, naturally led to the establishment of rich interactions, translated into questions, comments and sharing of ideas.

Finally, it is also important to highlight some transversal aspects stated by the pre-service teachers in the interviews. They did not recognize letter correspondence (pen pal exchange) as a possible strategy to develop mathematical communication and to apply evaluative writing/feedback. After this experience, they valued the potential of this approach and some considered using strategies of the same nature in the near future, such as journals or diaries, mainly due to the emphasis they give to written communication, which is poorly valued in the mathematics class. In addition, they found that this aspect of communication is somewhat complex for students due to the lack of regular experiences at this level. As future teachers, they became more aware of the importance of feedback, mentioning that it should be “concrete”, “accurate”, “clear”, but “not too revealing, as it is not intended to present the answer but guide the students’ activity”. Finally, they emphasized that the exchange of correspondence by letter enhances the engagement and motivation of the participants, helping them maintain interest and concentration levels in the mathematical activity being developed.

Conclusion

With this study we realized that a pen pal exchange (correspondence by letter) facilitates written mathematical communication and promotes effective engagement of the participants, through a genuine and contextualized interaction (e.g. Crespo, 2003; McCaffery, 2012; Norton & Katsberg, 2012). We also believe that the dynamics underlying this strategy and its dialogical nature can lead to more intentional and reflective communication/feedback in the sense that there is a real recipient and time to think carefully about the answer/comments. Letters were a vehicle to share ideas, access information/mathematical knowledge, formulate questions/tasks, naturally promoting the development of written communication, recurring to natural and mathematical languages presupposing an articulated and organized discourse (e.g. Crespo, 2003; Norton & Katsberg, 2012; Pugalee, 2015). They also allowed future teachers to put into practice their evaluative writing/feedback skills by sending comments in a meaningful context with a real recipient in mind.

This was a widely valued experience by the pre-service teachers who were able to access the thinking of students with whom they established communication, giving them diversified feedback in the context of evaluative writing. They sought to adapt and personalize the comments to each student they contacted with, highlight positive aspects of their work, provide information to guide them in further reflection, formulate clues for future action, and to reinforce the importance of improving certain aspects such as clarity and correctness of writing (Santos & Semana, 2015). In the letters they were able to include information about task performance

and the processes used, but also self-regulatory information and praises related to the self-level (Hattie & Timperly, 2007). We also have to state that some of the feedback was not always descriptive or incisive or even informative, as it did not provide clear or evident information about what students were doing well or had to improve. Despite this study being focused on future teachers, we can say, based on the content of the letters exchanged, that the students involved in the pen pal experience benefited from this experience. There were aspects highlighted in the written feedback that contributed to improve their knowledge of specific mathematical topics, mainly the ones involved in the tasks posed. The information provided to students by the pre-service teachers was sent with the intention of guiding and improving their mathematical learning, modifying, contributing to adjust their thinking (Black & William, 2009; Chong, 2018). In addition, it was also stressed in the letters that it is important in written mathematical communication to clarify the reasoning behind the solution of a task, which made these students more aware of the need to refine this mathematical ability.

To conclude, the letter correspondence provided these future teachers with a new lens on aspects such as written communication and feedback. Through the direct contact with a real recipient, they could see the importance of creating opportunities for students to communicate their thinking and the role of evaluative writing in self-regulation and consequently in learning. The possibility of putting into practice many of the contents covered in the unit course they were attending (Didactics of Mathematics), allowed them to value these aspects and mobilize them with greater appropriation.

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Author Information

Ana Barbosa

 <https://orcid.org/0000-0002-6314-7080>

Instituto Politécnico de Viana do Castelo

Rua Escola Industrial e Comercial Nun'Álvares, 34
4900-347 Viana do Castelo

Portugal

Contact e-mail: anabarbosa@ese.ipvc.pt

Isabel Vale

 <https://orcid.org/0000-0001-6155-7935>

Instituto Politécnico de Viana do Castelo

Rua Escola Industrial e Comercial Nun'Álvares, 34
4900-347 Viana do Castelo

Portugal