International Journal on Social and Education Sciences
Volume 2, Issue 1, 2020 ISSN: 2688-7061 (Online)

Examining Curricular Coverage of Volume Measurement: A Comparative Analysis

Dae S. Hong
University of Iowa, United States, dae-hong@uiowa.edu

Kyong Mi Choi
University of Virginia, United States

Cristina Runnalls
California State Polytechnic University, United States

Jihyun Hwang
University of Iowa, United States

Abstract: In this study, we examined and compared volume and volume-related lessons in two common core aligned U.S. textbook series and Korean elementary textbooks. Because of the importance of textbooks in the lesson enactment process, examining what textbooks offer to students and teachers is important. Since different parts of a textbook provide potentially different opportunities to learn (OTL) to students, we examined exposition, worked examples, and exercise problems as these three areas potentially provide OTL to teachers and students. We paid attention to various OTL that textbooks provide to teachers and students, the number of volume and volume-related lessons, students' development, learning challenges in volume measurement and response types. Our results showed that both countries' textbooks only provide limited attention to volume lessons, learning challenges are not well addressed, conceptual items were presented as isolated topics, and students often need to provide just short responses. We also made recommendations to teachers and textbook authors.

Keywords: Elementary textbooks, Volume Measurement, United States and Korean textbooks, Comparative Analysis

Introduction

Reform efforts in mathematics education call for using carefully selected tasks and reflecting students' developmental thinking in mathematics instruction (National Council of Teachers of Mathematics [NCTM], 2014). Among different resources that teachers can use, although researchers have different views, it is generally agreed that textbooks play an important role in shaping teachers' lessons (Polikoff, 2015; Remillard, Harris, & Agodini, 2014; Stein, Remillard, & Smith, 2007). Since textbooks are important resources in the lesson planning process, it is important to carefully design tasks in mathematics textbooks so that those tasks can be transformed to lessons that address students' developmental thinking, and promote reasoning and conceptual understanding. Limited coverage may decrease students' opportunities to learn (OTL) those topics (Smith, Males, Dietiker, Lee, & Mosier, 2013) and is a possible reason for students' difficulties in learning those mathematical topics.

Among the many topics in mathematics, we selected volume lessons to analyze in our study because findings from national assessments indicate that U.S. students perform poorly in the measurement domain compared to any other mathematical domain (Lehrer, 2003; Mullis, Martin, Foy, & Arora, 2012; Mullis, Martin, Foy, & Hooper, 2016). Previous studies demonstrate curricular limitations of popular U.S. textbooks in length and area measurement topics (Smith et al., 2013; Smith, Males, & Gonulates, 2016). In comparison, there are limited studies about volume measurement (Sarama & Clements, 2009; Vasilyeva et al., 2013). Furthermore, while researchers have examined other mathematical topics in textbooks (Smith et al., 2016; Son & Hu, 2016), volume lessons in textbooks have not.

Thus, our goal for this study is to examine OTL in two common core-aligned American elementary textbook series and Korean elementary textbooks. We chose Korean textbooks because Korean students do well on international comparative studies. In the U.S., there are several common core–aligned elementary mathematics textbook series with the development and implementation of Common Core State Standards of Mathematics (CCSSM). Many previous textbook analysis studies examined textbooks that were developed prior to the development of CCSSM (Otten, Gilbertson, Males, & Clark, 2014; Smith et al., 2013; Son & Hu, 2016). Thus, our analysis of measurement topics of volume in two popular common core-aligned American textbook series and Korean textbooks will expand our understanding of how textbooks treat measurement lessons. The following are our research questions:

1. How do American and Korean textbooks distribute attention to volume and volume-related lessons?
2. In what order do textbooks present volume and volume related lessons?
3. How well do textbooks address well-known challenges in learning volume measurement?

Related Literature

Teaching and Learning Volume Measurement and Fundamental Concepts in Volume

Students can learn volume measurement by coordinating several different ideas such as filling three-dimensional space with same–sized units, counting same–sized units, understanding and iterating layer structure, and linking layer structure to the volume formula (Battista, 2004; Sarama & Clements, 2009; Vasilyeva et al., 2013). These conceptual ideas are fundamentally important in learning measurement in general (Smith et al., 2013; Smith et al., 2016). However, these fundamental ideas are more challenging to students with the three–dimensional nature of volume measurement (Curry, Mitchelmore, & Outhred, 2006; Sarama & Clements, 2009). For example, Trends in International Mathematics and Science Study 2011 (TIMSS 2011) show that only 26% of U.S. 8^{th} graders answered item M052206 correclty, which requires students to use a same-sized unit (a book in this problem) repeatedly to estimate the volume of a box, correctly (Mullis et al., 2012).

Also, U.S. 8^{th} graders have difficulties compared to their Asian counterparts when solving item M032100 about being able to see and structure three-dimensional space that can be filled with unit cubes (Mullis et al., 2012); 60% of U.S. students answered this item correctly compared to 87% of Korean students. Similar results are demonstrated in previous studies, where students often struggle with counting the number of cubes in a three-dimensional structure correctly (they often count the faces of the cubes, which often leads to multiple counting) (Battista & Clements, 1996) and are not able to see the structure of cubes in three–dimentional layer (Battista, 2004).

Previous study shows that it is important for students to gradually understand how to structure, iterate, and link three-dimensional layer structure to volume measurement (Curry et al., 2006; Sarama & Clements, 2009). When students are able to understand space–filling ideas with same–sized units and layer structure, they tend to answer volume measurement problems correctly. However, without using space–filling and layer structure, they often use the volume formula without knowing why and how the formula works (Vasilyeva et al., 2013). These results indicate that space-filling, unit iteration, and linking units to the formula are key ideas to successfully understand volume measurement.

Three fundamnetal ideas in volume measurement are also described in leraning trajectory. Sarama and Clements (2009) comprehensively summarized research findings on learning mathematics and suggested detailed learning trajectories for various mathematics content areas. They described the learning trajectories of geometry and geometric measurement at great length, carefully outlining key characteristics of the different stages of understanding as well as what students may (or may not) be able to do at each stage in the process. Some foundational concepts for volume measurement include filling a given three–dimensional space with unit cubes, comparing by counting rows of arrays and understanding layer structure (Battista & Clements, 1996; Sarama & Clements, 2009). The learning trajectory of volume begin from the stage of recognizing, filling three - dimensional quantities with unit and end with the stage of structuring two and three- dimensional spaces (layer) systematically. Since textbooks are one of the important curriculum materials, it is critical to reflect and include mathematical tasks that address these fundamental ideas in volume. In fact, examining how textbooks' content reflect students' development steps in learning trajectories is one of important areas to consider when selecting textbooks (NCTM, 2014).

Korean students have similar challenges, where they are not able to count unit cubes correctly (Kim, 2013) and score the lowest on volume items (Lee, 2006). Despite these findings, there are limited studies about teaching and learning volume in Korea (Ha, Pang, & Ju, 2010). These studies reveal common learning challenges in volume measurement so it is important to provide learning opportunities to experience important ideas in volume measurement with carefully designed and selected tasks (Kim, 2013).

What Textbooks Offer in the Lesson Enactment Process

Researchers often refer to what textbooks offer as opportunities to learn (OTL) mathematics (Otten et al., 2014; Smith et al., 2013). OTL is often defined as whether or not students have had the opportunity to study a particular topic or learn how to solve a particular type of problem (Floden, 2002; Liu, 2009). In terms of what textbooks offer to teachers and students, these questions lead us to consider the number of textbook pages that cover certain mathematics topics, how those textbook pages cover those topics (content coverage and exposure), and how teachers use those textbook pages to plan their lessons (instructional delivery) (Liu, 2009).

In the early stages of the lesson enactment process, teachers use textbooks and other curriculum materials to select and possibly modify tasks and activities that are appropriate for particular lessons (Remillard, 2005; Remillard & Heck, 2014). Teachers will select and modify tasks and activities based on their belief and pedagogical orientations (Roth McDuffie, Choppin, Drake, Davis, & Brown, 2017; Son & Kim, 2015). It is certainly possible that some tasks and activities in textbooks may not be selected. However, both U.S. (70 to 98 %) and Korean teachers use textbooks often to plan their lessons (Chingos & Whitehurst, 2012; Pang, 2014). After the lesson planning stage, when teachers enact those lessons, what is included from the textbook can transform into OTL for students. However, if the textbook provides limited attention to volume measurement topics and fundamental concepts in volume measurement, it is possible that the teachers' lessons will have limited coverage of those topics and challenges as well. Furthermore, if the textbook presents volume lessons in different orders compared to possible students' learning development in volume measurement, it is possible that teachers' volume measurement lessons might not reflect students' learning development well. As a result, these issues of coverage in textbooks can lead to lacking or limited OTL for students to learn those topics because they may not be well reflected in the teachers' lesson plans. It is natural to think that increased OTL can lead to better possibility for students to answer certain mathematical problems correctly (Floden, 2002).

Additionally, how textbooks and curriculum materials present and cover mathematical topics will be an important coverage issue as well. Researchers categorized curriculum materials as *delivery mechanism* (DM) and *thinking device* (TD) (Roth McDuffie et al., 2017). DM curriculum materials were designed from a monologic function, geared primarily toward procedural mastery and content delivery with limited opportunities for students to express and wrestle with their thinking while TD curriculum materials consider students' learning trajectory, give students opportunities to express and wrestle with their thinking to learn mathematics conceptually (Roth McDuffie, Choppin, Drake, & Davis, 2018). When teachers are using TD curriculum materials, it is more likely to implement effective teaching strategies recommended by NCTM (Roth McDuffie et al., 2017). On the other hand, when teachers are using DM curriculum materials, teachers are more likely to implement mathematics lessons with limited attention to effective teaching practices (Roth McDuffie et al., 2018; Roth McDuffie et al., 2017). In all, what textbooks offer to teachers and students can have impact on teachers' mathematics lessons. If textbooks place more attention to procedures and have limited attention to concept and opportunities to express students' thinking, it is possible that their lessons plans also lack those learning opportunities.

Methods

Data Sources

A number of elementary mathematics textbooks are available in the United States, which has no national curriculum. Thus, we were forced to choose some series over others for analysis. Two textbooks series that we selected - *Go Math* and *MyMath* - are Common Core-aligned textbooks from two major U.S. publishers (Houghton Mifflin and McGraw-Hill, respectively). These are two popular elementary textbook series (Sahm, 2015) and, according to McGraw-Hill, there are 3 million *MyMath* users in the United States. Two recent studies examined length and area lessons in other non-Common Core American textbooks and several other studies examined textbooks that were developed before the introduction of Common Core State Standards (CCSS) (Charalambous, Delaney, Hsu, & Mesa, 2010; Son & Senk, 2010). Therefore, examining these common core aligned textbooks series may expand our understanding of how more current American textbooks treat volume measurement.

We examined textbooks from grade 1 through grade 6 from each publisher. We examined all volume and volume related lessons found in these searches, up to the initial introduction of the volume formula, $V = \text{length} \times \text{width} \times \text{height}$. All textbooks were the latest editions available, published in 2014 or 2015. In all, 275 *GoMath* and 175 *MyMath* items were analyzed. Items included exposition, worked examples, and exercise problems.

Korea has a national curriculum with a centralized educational system, where the government develops and provides guidelines for the school curriculum. Korean textbooks are either published by the government or are authorized by the government. Elementary School Mathematics, the Korean textbooks we examined in this study, are published by the government and the only mathematics textbooks used in elementary schools in Korea because all Korean elementary textbooks are published by the government. All Korean textbooks from grade 1 to 6 were examined similarly as the U.S. textbooks. Volume and volume-related topics were seen in grades 1, 3, 4, and 5 textbooks. These Korean textbooks are the latest editions published by the Ministry of Education in Korea. Table 1 describes the number of pages and lessons examined in this study.

Table 1. Textbooks used in the Study

Textbook Series	Publisher	Publication Date	Number of Pages	Number of Items	Number of Lessons
GoMath series	Houghton Mifflin	2015	104	275	12
MyMath series	McGraw-Hill	2014	28	175	7
Korean Textbooks	The Ministry of Education in Korea	2014, 2015	36	116	12

Framework of Analysis

Different frameworks are used to analyze textbooks (Cady, Hodges, & Collins, 2015; Smith et al., 2016; Son & Hu, 2016). Some studies examined cognitive demand of tasks (Charalambous et al., 2010; Hong & Choi, 2018), pedagogical orientations (Cady et al., 2015; Nie, Cai, & Moyer, 2009; Son & Senk, 2010), representations used in textbooks (Son & Senk, 2010) and how challenging mathematical tasks are (Son & Hu, 2016; Zhu & Fan, 2006). As previously described, curriculum materials can be categorized as DM and TD (Roth McDuffie et al., 2017). Additionally, volume measurement research studies indicated several fundamental concepts and those concepts are also demonstrated in learning trajectory (Battista & Clements, 1996; Sarama & Clements, 2009; Vasilyeva et al., 2013).

Furthermore, there have been several studies that showed measurement lessons are often procedural rather than conceptual (Baturo & Nason, 1996; Murphy, 2012). When measurement lessons in textbooks are examined, fundamental concepts, learning trajectories, procedural and conceptual knowledge types were examined (Hong, Choi, Runnalls, & Hwang, 2018, 2019; Lee & Smith, 2011; Smith et al., 2013; Smith et al., 2016). Thus, we examined these different views on curriculum materials, previous volume and textbook analysis studies to frame our analysis (Table 2). Since textbooks are composed of several different parts, each of which provide potentially different OTL to students, researchers use the term *knowledge in pieces* to describe OTL in textbooks (Smith et al., 2013; Smith et al., 2016), meaning students' learning is the collection and coordination of many different elements. Teachers and students can collect and coordinate the different parts in textbooks to plan lessons and use those parts to study on their own.

In previous studies, researchers often examined exposition (e.g. introductory paragraphs, text boxes with definitions, formulas, or theorems), worked examples (problems presented together with an explained solution), and exercise problems (mathematical items students are expected to solve) because they provide potentially different learning opportunities for students (Cady et al., 2015; Son & Hu, 2016). Exposition can be used to introduce mathematics content including definitions and formulas (or students can read exposition on their own to study). Worked examples demonstrate how certain mathematics problems are solved where students have opportunities to engage in mathematical tasks with exercise problems. Students become familiar with how the volume formula is used and practice on their own with worked examples and exercise problems. Finally, teachers use all three areas to plan their lessons and implementing and enacting those lessons lead to students' opportunities to learn those topics in their classrooms. Table 2 describes our framework for this study. We will describe each focus in detail.

Table 2. Analysis Framework of Content and Problems

Area of Focus

• Number of volume and volume-related lessons
• Timing and topic sequence
  ○ Lesson Titles
  ○ Learning Trajectory
• Procedural and conceptual knowledge
• Fundamental Concepts in learning volume measurement
  ○ Filling space equal-sized units
  ○ Layer structure
  ○ Volume formula and definition

The Number of Volume and Volume-related Lessons

To understand how much attention is given to volume lessons, we simply determined the number of lessons related to volume topics. We first examined the table of content of each textbook to identify lessons that have the term "volume" in the title. We then conducted a manual search to locate those lessons that do not have volume in the title but are related to volume topics. Those topics include space–filling, layer structure, and building three–dimensional structure. To include every lesson that has to do with volume, we included all textbook pages that had at least one instance of such content. The total number of volume and volume-related lessons was compared to the total number of lessons in each textbook to measure how much attention was given to area measurement. The search for these lessons enabled us to see if these textbooks provide any prior and necessary knowledge for topics related to volume measurement.

Timing and Sequence

When learning mathematics, we often think about prior knowledge that students need to solve mathematics problems. This is especially important in measurement topics. Particularly for measurement topics, many researchers have pointed out that students gradually learn related topics (Battista, 2004; Outhred & Mitchelmore, 2000; Sarama & Clements, 2009) and according to volume learning trajectory, there are different skills, such as filling three–dimensional space with equal–sized units and layer structure, which are often challenging to students before a certain age (Battista, 2004; Sarama & Clements, 2009). It is recommended that these fundamental topics be introduced to students before introducing the volume formula $\text{length} \times \text{width} \times \text{height}$. A lack of these early topics in textbooks may lead to missing those topics in teachers' lesson plans and teachers' lessons that do not reflect students' development in learning volume measurement, which can keep students from experiencing and learning those topics, limiting their OTL to gradually experience them. We examined how each textbook series presented volume measurement in regards to research recommendations for timing and sequence of area topics.

Procedural, Conceptual, and Conventional Knowledge

When examining textbook items, researchers often consider the procedural and conceptual knowledge required for each item (Smith et al., 2016; Son & Senk, 2010). One of the effective teaching practices recommended by NCTM is to build procedural fluency from conceptual understanding (NCTM, 2014). Furthermore, NCTM (2014) also recommends including more meaningful tasks that promote reasoning and problem solving. In attempts to enact these effective teaching practices, it is beneficial to balance procedural and conceptual items in textbooks. Conceptual knowledge is defined as "knowledge that is rich in relationships" that cannot exist as "an isolated piece of information" while procedural knowledge is defined primarily as knowledge of sequential quality (Hiebert & Lefvre, 1986). Conceptual knowledge in volume may be seen as recognizing that the volume of a three-dimensional space is defined as the number of unit cubes that completely cover that three–dimensional space and understanding and linking layer structure to the volume formula. Procedural knowledge in volume measurement can be sequences of steps that are sufficient to solve volume problems, including using the volume formula and counting unit cubes. Classifying these two types of knowledge is helpful in understanding how textbooks treat well-known conceptual challenges and difficulties, as well as their focus.

We also included a type of knowledge called "conventional" (Smith et al., 2016). This knowledge type represents choices about how to write certain terms in volume measurement, such as the definition for a unit cube. Categorizing textbook items into knowledge types can indicate the curricular focus of the textbooks we compare. Placing more attention on procedures can limit students' opportunities to learn area concepts and lead students to depend more on procedure.

Fundamental Concepts in Understanding Volume Measurement

As previously mentioned, we were able to identify fundamental concepts in volume measurement. We were interested in knowing whether selected textbooks provide OTL to students so students can be exposed to those important concepts. We identified several fundamental concepts in volume measurement: Space–filling with same–sized units, layer structure, and linking layer structure to the volume formula. When these fundamental concepts have limited coverage, teachers may not be able to reflect these challenges in their lessons (or students may not experience these topics if they studied on their own) which, in turn, can limit students' OTL.

Coding Procedures and Examples

As previously mentioned, exposition, worked examples, and exercise problems provide potentially different OTL to students. When textbooks are analyzed, the frequency or the number of times a particular topic appears on textbook pages are often computed. These frequencies are usually weighted equally (Ding, 2016; Polikoff, 2015; Smith et al., 2016) because it is the most logical and defensible approach as it is too challenging to decide which items would give more meaningful OTL to students. Next, we needed to decide on the unit of analysis. Figure 1 shows one example of unit of analysis we used in this study.

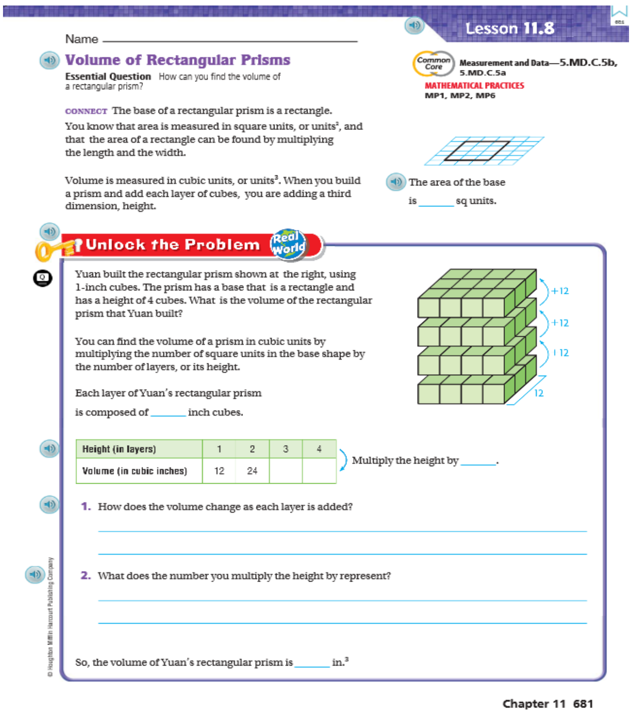


Figure 1. An Example showing Unit of Analysis (*Go Math*, 2015 d, p. 681)

Since each exposition, worked example, and exercise problem in textbooks has its own instructional purpose (potentially different OTL), we first considered each worked example, exercise problem, and exposition as one unit of analysis as each item provides OTL to teachers and students. Figure 1 is a sample page from one of the textbook series. This page includes one exposition, one (partially) worked example, and two exercise problems – there are four items (four units) to analyze. The first item, exposition, is about defining the volume using layer structure. The (partially) worked example demonstrates how layer structure and volume are related. With the worked example, students see how adding one layer can change the volume. Finally, two exercise problems allow students to see how layer structure and volume are related. For all other textbook pages, we used the same method to identify expositions, worked examples, and exercise problems to count the number of units to be coded from each page. We will provide more coding examples in detail to describe how we identified and coded each item. Once we agreed on our unit of analysis, the authors met several times to understand the analytic framework, including developing an understanding of procedural and conceptual knowledge, and students' challenges in volume measurement. We then assigned codes to each identified textbook item. Figures 2, and 3 show examples of how we coded each item.

Translate: Compute the volume of rectangular prism

Figure 2. Coding Examples from a Korean Textbook (The Ministry of Education in Korea, 2015, p. 187)

First, we decided that there is one unit (one exercise problem) to be coded. Students are asked to compute the volume of a rectangular prism. This item is coded as finding volume using the formula and procedural because students just need to multiply three numbers to get the volume. Figure 3 shows one item (one exercise problem or one unit) to code. This item was coded as counting unit cubes without layer structure because it is about counting the number of cubes in the picture. Although students may use layers to count the number of cubes and gain conceptual understanding, it was not specified in this problem. When this task is included in *Go Math* series, the idea of layer was not introduced to students yet (linking the number of cubes in each layer is introduced in 5th grade textbook). Without paying attention to layer structure, students might not be able to count the cubes correctly as previous studies indicated (Battista, 2004; Battista & Clements, 1996). Thus, we differentiated these items compared to the item in Figure 4. It was coded as procedural knowledge because students simply need to count the cubes without linking to the formula or considering layers.

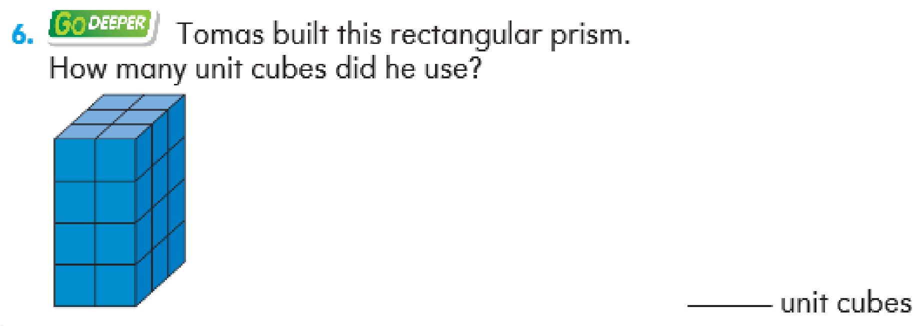
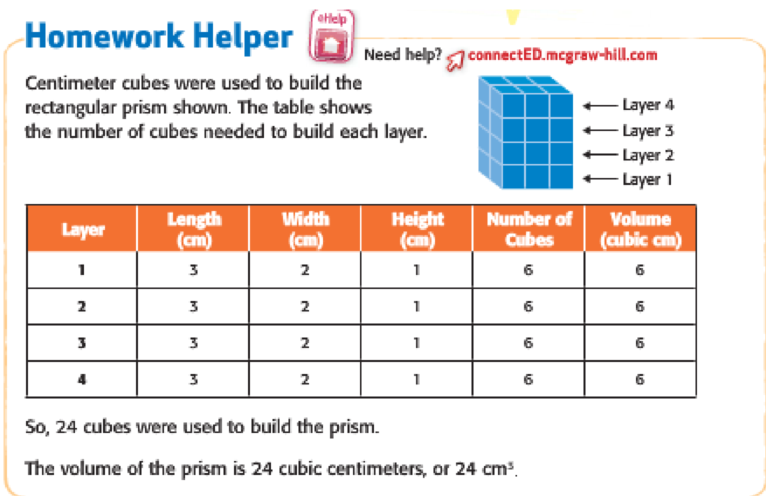


Figure 3. Coding Example from one U.S. Series (*Go Math*, 2015 a, p. 720)

There was one item (one exposition or one unit) to code in Figure 4, as it is about one topic, namely layer structure. It describes what happens to the total volume when layers are added to the structure. It was coded as conceptual, as it outlines the process of using layer structure to understand volume.



Layer	Length (cm)	Width (cm)	Height (cm)	Number of Cubes	Volume (cubic cm)
1	3	2	1	6	6
2	3	2	1	6	6
3	3	2	1	6	6
4	3	2	1	6	6

So, 24 cubes were used to build the prism.

The volume of the prism is 24 cubic centimeters, or 24 cm^3.

Figure 4. Coding Example from one U.S. Series (*MyMath*, 2015 b, p. 953)

Reliability

After careful discussion of analytic framework, codes and unit of analysis, the two authors coded approximately 20% of the textbook items to check inter-rater reliability. At most three codes are given to each item because there are two possible codes (i.e. procedural vs. conceptual, knowledge needed) for each item. After initial coding by the two coders and finding an acceptable high inter-rater reliability, the authors coded all textbook items jointly to produce a final set of tables for analysis, resolving coding differences of individual items when they arose. To determine reliability, we applied a generalizability theory D study (Alkhrausi, 2012). This technique produced a reliable coefficient of 0.964.

Results

Volume Measurement Lessons in Textbooks

Table 3 shows the number of volume and volume-related lessons in the textbooks. Both U.S. and Korean textbooks place limited attention to volume and volume-related lessons. Each series included between .9 % to 10.8 % of lessons focused on volume and volume-related topics. These percentages are less than 3% to 13% found in length and area measurement lessons (Smith et al., 2013; Smith et al., 2016). This finding indicates that neither American nor Korean textbooks place much attention on volume lessons. More volume lessons were found in the 6th grade Korean textbook.

Table 3. Number and Percentage of Volume and Volume-related Lessons to Total Lessons

	Grade	Volume and Volume-Related Lessons	Total
Go Math	2	1(0.9%)	110
	3	2 (1.9%)	105
	4	1(1.0%)	103
	5	8 (7.7%)	103
MyMath	3	1 (0.9%)	113
	4	1 (0.9%)	116
	5	5 (3.9%)	129
Korean Textbooks	1	1 (1.4%)	70
	3	4 (5.9%)	68
	6	7(10.8%)	65

Procedural, Conceptual, and Conventional Knowledge

Table 4 illustrates the percent distribution of different types of knowledge that each textbook has. Similar to length and area lessons in American textbooks, many items in volume lessons are procedural. In U.S. textbooks, conventional items were found often in lower grade textbooks because of the introduction of volume units.

Table 4. Percent Distribution of Conceptual, Procedural, and Conventional Knowledge in Textbooks

	Grade	Procedural	Conceptual	Conventional
Go Math	2	87.1	12.9	0
	3	71.3	0	28.7
	4	93.7	0	6.3
	5	90.2	9.8	0
My Math	3	81.2	0	18.8
	4	44.4	0	55.6
	5	82.1	16.9	1.0
Korean Textbooks	1	14.3	85.7	0
	3	44.8	2.6	52.6
	6	72.8	24.3	2.9

Time and Sequences

Table 5 displays the lesson topics and sequences of volume and volume-related lessons in the three textbook series. Both countries' textbooks introduce units of liquid volume such as liters and gallons in the early grades. We noticed two interesting lessons. First, in the second grade *GoMath* series, we found "Build Three-Dimensional Shapes."

In this lesson, we found an item in Figure 5. In Figure 5, the item is about the structure of a rectangular prism that is composed of unit cubes. Students have opportunities to see three–dimensional structure of layers from different views. Since students often struggle to see three–dimensional structure (Battista, 2004; Sarama & Clements, 2009), it is beneficial to include these items, but this was the only lesson that we found these items. Although students might be able to visualize layers when attempting to solve this problem, the layer ideas were not introduced to students when this problem was in included in *Go Math* series.

Table 5. Lessons and Topic Sequence of Volume Lessons in Three Textbooks Series

	Go Math	*My Math*	Korean Textbooks
Grade	Lesson Topic	Lesson Topic	Lesson Topic
1	• None	• None	• Comparing volumes (making visual comparative judgement)
2	• Build Three-Dimensional Shapes	• None	• None
3	• Estimate and Measure Liquid Volume • Solve Problems about Liquid Volume	• Solve Capacity Problems	• Comparing Capacity • Units of Capacity • Estimate and Measure Capacity • Sum and the difference of Capacity
4	• Metric Units of Liquid Volume	• Metric Units of Capacity	• None
5	• Metric Measures • Unit Cubes and Solid Figures • Understand Volume • Estimate Volume • Volume of Rectangular Prisms • Apply Volume Formula • Comparing Volumes • Find the Volume of Composed Figures	• Convert Customary Units of Capacity • Use Models to Find Volume • Volume of Prisms • Build Composite Figures • Volume of Composite Figures	• None
6	• None	• None	• Unit of Volume • Volume of Rectangular Prisms • Unit of Capacity • Building Three – Dimensional Shapes with blocks • Drawing Three – Dimensional Shapes From Different Views • Drawing Three – Dimensional Shapes • Connecting Cubes

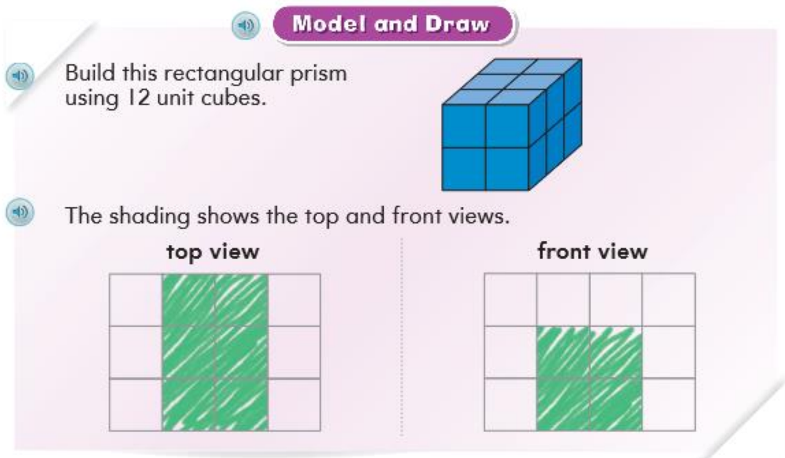


Figure 5. Item from *GoMath* that Explores Three–dimensional Shapes (*GoMath* 2015 a, p. 718)

The item in Figure 6 is the first time that the term "layer" was used in these textbooks. Understanding layer structure is challenging to students, so it may be beneficial for students to experience simple layer structure first (e.g. one or two horizontal or vertical layers) to become familiar with layer structure than experiencing multiple layer structure. Furthermore, students were not introduced how to link the number of cubes to each layer at this point (they would see it in 5th grade textbook). Thus, although beneficial, this item might cause some challenges for students to visualize layer structure.

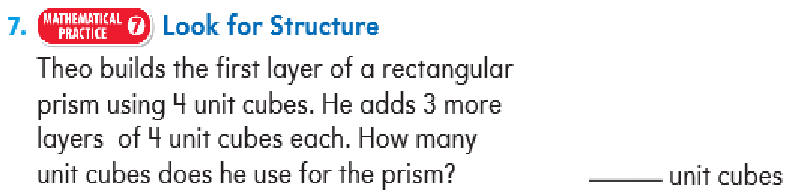


Figure 6. One Item that Uses "Layer" (*GoMath*, 2015 a, p. 720)

The first volume lesson that we found from the Korean textbook is about comparing liquid volume. This lesson was not found in the U.S. textbooks. This lesson gives students ideas about what volume is (amount of space in containers) and how to compare volumes (Figure 7). This is why 85% of items in the 1st grade Korean textbook are coded as "conceptual." As described earlier, two US series also included liquid volume items early; however, items in U. S. series were mostly about introducing volume units such as liters and gallons rather than knowing capacity. As the results, many items in U.S. series were coded as either procedural or conventional.

Translation: Compare the amount of water that can fill each container.

- Discuss different ways to compare the amount of water that can fill each container.
- Compare the amount of water that can fill each container.

Figure 7. One Item from a Korean Textbook (The Ministry of Education in Korea, 2014 a, p. 134)

An interesting finding from the Korean textbook is "Building Three–Dimensional Shapes with Cubes" in the grade 6 textbook. Although this is an important and useful item to understand three–dimensional structure, this lesson was found after introduction of the volume formula. According to volume learning trajectory, students can attempt to fill three dimensional space at age 7 (normally grade 2) and count cubes in three – dimensional structure at age 8 (normally grade 3) (Sarama & Clements, 2009). At grades 2 and 3, students might not be able to answer volume tasks correctly; however, sudden introduction of the volume formula with limited experiences of three fundamental ideas might lead students to using the volume formula without knowing why. These results from volume learning trajectory indicate that including these fundamental items in textbooks early will provide OTL for students to get familiar with these ideas. This issue of time and sequence can lead to the previously mentioned TIMSS 2011 results for 8th graders (items M052206 and M032100).

Fundamental Concepts in Understanding Volume Measurement

Table 6 shows the distribution of each volume–related knowledge type presented in American and Korean textbooks. We describe each type in more detail below.

Filling Three-dimensional Space with Same-sized Units

Being able to filling three-dimensional space with equal–sized units is important knowledge for students to understand the volume formula. If students are able to partially fill three–dimensional space with same–sized units, they have a composite unit to fill remaining space (Vasilyeva et al., 2013). Iteration of a composite unit can lead to successfully understanding volume measurement. Filling three-dimensional space can be introduced with unit cubes or any other three-dimensional units such as a book (similar to Figure 2). The *GoMath* series and Korean textbooks contain space- filling items in the 5th and 6th grade textbooks. Although fundamentally important, there are only a few items related to filling three-dimensional space with same–sized units (about 10%). Also, these items were included after an introduction of the volume formula. Although it can be meaningful to students who can use the volume formula to learn space–filling ideas to understand and link three-dimensional structure to the volume formula, filling three-dimensional space items should be introduced earlier. At an early age, it may be challenging for students to understand filling three-dimensional space, but sudden introduction of the volume formula without a conceptual idea of space–filling can lead them to use the formula without understanding why it works.

Table 6. Percent Distribution of Knowledge Type in Textbooks

	Grade	Space-Filling	Building Three-Dimensional Structure	Counting Cubes (without layer structure)	Layer Structure	Volume Definition and Formula	Volume Units	Other Topics
Go Math	2	0	80.6	6.4	12.9	0	0	0
	3	0	0	0	0	0	100	0
	4	0	0	0	0	0	100	0
	5	9.8	0	28.9	12.3	22.2	1.0	25.8
My Math	3	0	0	0	0	0	100	0
	4	0	0	0	0	0	100	0
	5	0	0	17.8	13.2	48.1	1.3	19.6
Korean Textbooks	1	0	0	0	0	0	0	100
	3	0	0	0	0	0	100	0
	6	10.0	24.2	31.4	0	28.6	2.9	2.9

Previous studies demonstrated that some 5th grade students are not able to figure out the number of cubes that can fill a box (Battista, 2004; Battista & Clements, 1996). These studies show that it is important for students to be exposed to important conceptual ideas of filling three- dimensional space. In Figure 8, students use a composite unit (one layer of books) to estimate volume. Tasks like this can provide meaningful activity for students in understanding iteration of one layer to understand the volume formula. For example, being exposed to items like in Figure 8 would better prepare students to answer TIMSS 2011 item M052206 but there are only few items about space – filling ideas.

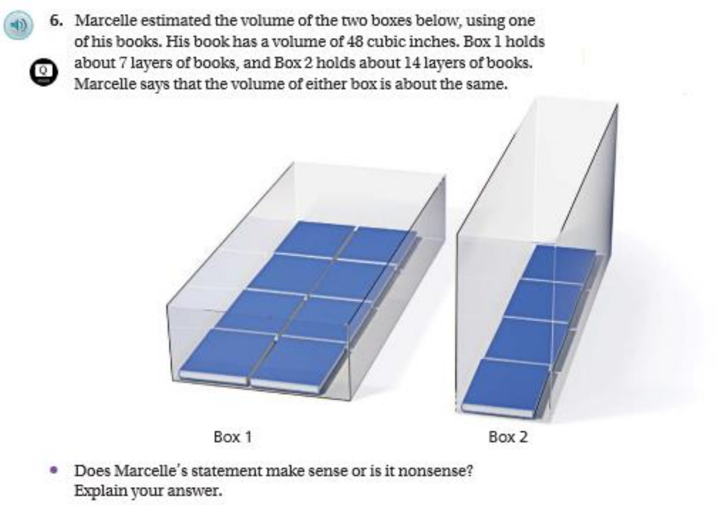
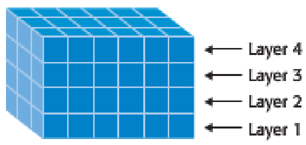


Figure 8. Space-filling Item from *GoMath* (*GoMath*, 2015 c, p. 678)

Layer Structure

Being able to use and see layer structure is another important conceptual idea in understanding volume measurement. If students are able to count unit cubes in each layer in a three-dimensional space, they can use a partial layer structure to have composite units and then use iteration or skip counting to measure the volume of the figure (Battista, 2004; Vasilyeva et al., 2013). Being able to structure three-dimensional space with layers can lead students to link the number of cubes in the rows and columns to the volume formula. Items in this category include counting unit cubes in each layer (The term "layer" should be included for coding in this category). Like space-filling items, items related to layer structure are also included 5th grade textbooks. Only *GoMath* introduces this in 2nd grade. In all three series, these items are included between 0% to 13.2%. For example, Figure 9 shows how layer structure can lead us to the volume of a rectangular prism. Students link unit cubes in each layer to the length, width, and height of the rectangular prism, allowing them to meaningfully link layer structure to the volume formula. However, only a limited number of layer items were found in these textbooks. The Korean textbooks do not have any items related to layers of three-dimensional space. Furthermore, by experiencing items about layer structure, students can be better prepared to answer TIMSS item M032100 but students were not exposed to these items often.

Use centimeter cubes to build the rectangular prism shown.



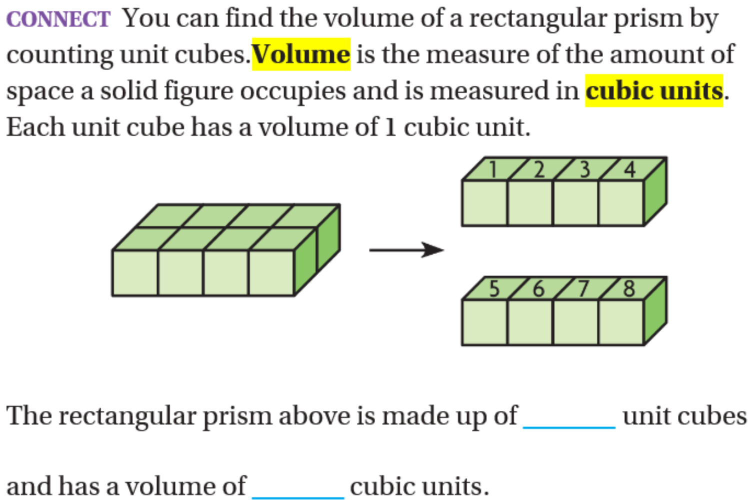
7. Complete the table below.

Layer	Length (cm)	Width (cm)	Height (cm)	Number of Cubes	Volume (cubic cm)
1					
2					
3					
4					

Figure 9. Layer Structure Item from *MyMath* Series (*MyMath*, 2014 c, p. 951)

Volume Definition and Formula

Volume definition and formula are introduced in 5th and 6th grade textbooks and the definition is found in an exposition section. Items were coded as volume formula if they showed that multiplying three numbers gives the volume, or if they used the *length* $\times$ *width* $\times$ *height* formula to compute the volume. In the U.S. series, either counting unit cubes or counting unit cubes in layers were used to define the volume of rectangular prism (Figure 10).

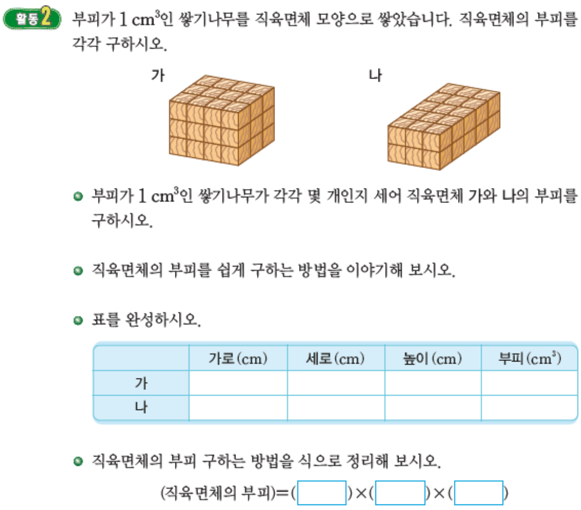

CONNECT You can find the volume of a rectangular prism by counting unit cubes. **Volume** is the measure of the amount of space a solid figure occupies and is measured in **cubic units**. Each unit cube has a volume of 1 cubic unit.

The rectangular prism above is made up of ______ unit cubes

and has a volume of ______ cubic units.

Figure 10. Item using Unit Cubes to Introduce the Volume Concept (*GoMath*, 2015 c, p. 669)

The Korean textbook does not give a definition but uses unit cubes in rectangular prisms to derive the volume formula (Figure 11).


활동 2 부피가 1 cm^3인 쌓기나무를 직육면체 모양으로 쌓았습니다. 직육면체의 부피를 각각 구하시오.

- 부피가 1 cm^3인 쌓기나무가 각각 몇 개인지 세어 직육면체 가와 나의 부피를 구하시오.
- 직육면체의 부피를 쉽게 구하는 방법을 이야기해 보시오.
- 표를 완성하시오.

	가로 (cm)	세로 (cm)	높이 (cm)	부피 (cm^3)
가				
나				

- 직육면체의 부피 구하는 방법을 식으로 정리해 보시오.

(직육면체의 부피)=( )×( )×( )

Translation: With unit cubes, rectangular prisms were built. Find the volume of rectangular prisms.

- Find the volume by counting the number of unit cubes.
- Think about how to find the volume.
- Complete the table.

	Length	Width	Height	Volume
Ga				
Na				

- Write the formula for the volume of a prism
  (volume of rectangular prism) = ( ) × ( ) × ( )

Figure 11. Introduction of the Volume Formula in a Korean Textbook (The Ministry of Education in Korea, 2015, p. 186)

As previously mentioned, only a limited number of space–filling and layer structure items are included in both countries' textbooks. Therefore, it is likely challenging to link the number of cubes in each layer to the volume formula, even if layers were used in some items. With limited OTL to explore important conceptual ideas in volume measurement, students are likely to resort to more procedural approaches to volume. We can see such a tendency in each textbook, as they all move quickly to items like the one in Figure 12, where students are only required to use a procedure of multiplying length, width, and height.

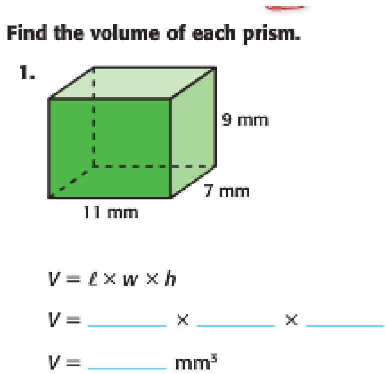
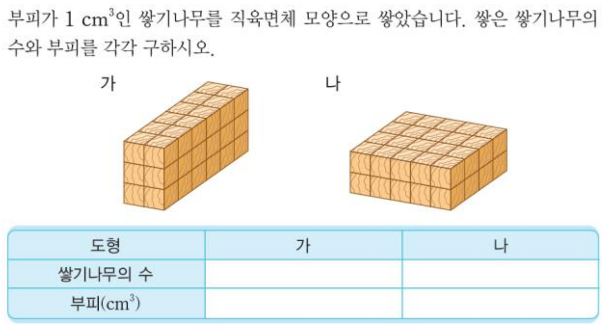


Figure 12. Simple Volume Formula Item from *MyMath* (*MyMath*, 2014 c, p. 956)

Other Volume Topics

Both U.S. and Korean textbooks have several items about counting unit cubes in rectangular prisms (Figure 13). There are more items about counting cubes compared to space–filling and layer items (17.8% to 31.4%). Although these items are useful, limited space–filing and layer items make it challenging to count the number of cubes correctly. It is possible that students will attempt to count unit cubes procedurally without knowing the concepts behind them. Previous studies showed students often count the faces of cubes, which can lead to double or multiple counting (Battista, 2004; Vasilyeva et al., 2013).



Translation: Two rectangular prisms were built with unit cubes. Find the number of unit cubes in each prism and find the volume.

Three-Dimensional Shapes	Ga	Na
The number of unit cubes		
Volume		

Figure 13. Item on Counting Cubes in a Korean Textbook (The Ministry of Education in Korea, 2015, p. 183)

A more frequent item was building three–dimensional structure. These items were found in the 2nd grade *GoMath* series and 6th grade Korean textbooks. Although these items are useful, these lessons were isolated lessons without attempts to link building three-dimensional shapes to other important volume concepts. These lessons and items were not found in other lessons in the three textbook series.

Summary and Discussion

In this study, we examined and compared volume and volume-related lessons in two common core aligned U.S. textbook series and Korean elementary textbooks. We examined exposition, worked examples, and exercise problems as these three areas potentially provide different OTL to teachers and students. We specifically paid attention to the number of volume and volume-related lessons, students' development, and learning challenges in volume measurement. Our results showed that both countries' textbooks only provide limited attention to volume lessons and that fundamental volume concepts are not well addressed in these textbooks. Our results show similar tendency in measurement lessons that was found in both area and length measurement lessons

(Smith et al., 2013; Smith et al., 2016), where there is limited attention in those lessons, mostly procedurally focused, and fundamental concepts in measurement are not well addressed in textbooks. In volume measurement, filling three-dimensional space with same–sized units, understanding layer structure, and linking layer structure to volume formula are key conceptual ideas in understanding volume measurement (Battista, 2004; Vasilyeva et al., 2013). However, these fundamental concepts are not well addressed in either country's textbooks, where fundamental concepts are covered late with just a limited number of items. We found more items about counting unit cubes without placing much attention to space–filling or layer structure. Also, some potentially important conceptual items related to space – filling and three – dimensional structure are presented as isolated topics without being connected to other important volume topics. Not being exposed to these fundamental ideas in volume measurement can be one of the reasons why U. S. students' struggle in learning volume measurement. For example, including more items like ones in Figures 4 and 8 can increase students' OTL and possibly increase students' chance to answer similar items in TIMSS that we mentioned previously.

Previous study shows that students learn measurement topics gradually (Sarama & Clements, 2009). Thus, we paid attention to time and sequence of the presentation of volume topics to understand whether three textbook series present volume lessons in ways that reflect students' learning development in volume measurement. We also found issues with time and sequence of the presentation of volume topics in the textbooks. All three series begin with liquid volume measurement, but only the Korean textbook include the idea of comparing liquid volume of containers (Figure 7). Such items provide students with opportunities to understand what volume means. After addressing liquid volume and operations involving liquid volume and capacity in the early grades, both countries' textbooks use unit cubes or equal–sized units and layer structure to introduce the volume definition and the formula $\text{length} \times \text{width} \times \text{height}$ in grade 5 (U.S.) and grade 6 (Korean) (Figures 10, 11). However, prior to introducing the definition and the formula, only limited attention is given to space–filling and layer structure. All three series move quickly to items like the one in Figure 12. This shows that all three series are more procedurally focused in volume lessons. This again confirms results from previous studies (Smith et al., 2013; Smith et al., 2016). A frequent finding in measurement research is students' tendency to rely on formulas without knowing why they work (Vasilyeva et al., 2013; Zacharos, 2006). Our study confirms that textbooks' attention placed on procedures may lead students to rely more on the volume formula without knowing why. Although procedural fluency is an important in learning mathematics, with the ways that these textbooks present volume measurement topics, it will be challenging to build procedural fluency from conceptual understanding.

Conclusions

With OTL these textbooks provide, both countries' textbooks can be classified as DM rather than TD – OTL offered by these textbooks offer to teachers and students can cause limited time spent on volume lessons that are procedurally focused with limited attention to important fundamental concepts and learning trajectory, and providing limited opportunities to express students' thinking. The results of our textbook analysis are not the only reason for students' difficulties in learning volume measurement, but what these textbooks offer to teachers and students can cause limited time spent on volume lessons that are procedurally focused without much attention to important learning challenges. As a result, students' OTL to learn volume measurement conceptually are limited and limited OTL can reduce the possibility of answering volume related tasks (e. g. those TIMSS 2011 items) correctly. With these textbook limitations, it is challenging for teachers to include meaningful tasks that promote reasoning and conceptual understanding and challenging for students to build procedural fluency from conceptual understanding. Teachers may need additional support to implement effective practices recommended by the NCTM (2014). Thus, to inform teachers about curricular limitations that we found, we can provide professional development to teachers so they can plan their lessons with modified tasks to fill the gaps and address learning challenges in volume measurement. For example, we recommend including more items like those in Figures 8 and 9 for students to become familiar with space–filling ideas and layer structure.

In terms of international comparative studies, we are not able say that the learning opportunities these textbooks offer are directly related to U.S. students' performances in TIMSS. However, our results show that the way these textbooks introduce and treat volume measurement can be one of the reasons for the U.S. students' TIMSS results. While we can say textbooks can be one of the reasons for U.S. students' performances, it is challenging to understand Korean students' results. Despite the fact that Korean textbooks shared many of the similar limitations as the U.S. textbooks, Korean students performed equally well in measurement compared to other content areas in TIMSS 2011and 2015 (Mullis et al., 2012; Mullis et al., 2016). At the same time, Korean students have similar struggles as U.S. students (Lee, 2010). This leads us to consider the results of TIMSS and

PISA more carefully (Wang, Barmby, & Bolden, 2017). Other mathematics education researchers noted that East Asian students are superior in computational and procedural problems but their performances are relatively similar to US students on open-ended items (Cai, 2005; Cai & Nie, 2007). Furthermore, many items in TIMSS and PISA are multiple choice or short response items (over 80 % of TIMSS and 60% of PISA items, respectively) (Neidorf, Binkley, Gattis, & Nohara, 2006). As we found, with many procedural items in Korean textbooks, it is possible that Korean students' high overall achievement may be more about doing well on multiple choice and short response items. For future research, it will be interesting to examine the results of TIMSS and PISA of students' performances on different item types. Results from these further studies may enable us to have a better understanding about Korean students' international assessment results.

One important finding from the current study is the alarming pattern we found in the coverage of measurement topics. Conceptual limitations in measurement lessons are commonly found in length and area measurement lessons (Smith et al., 2013; Smith et al., 2016) and now in volume measurement lessons. It will be important to examine more textbooks from other countries to understand if we can find similar pattern in those textbooks or not. In addition to informing teachers of these limitations, textbook authors and publishers need to be aware of these limitations so that these important topics can be reflected better in textbooks. When textbook authors and publishers are informed of these limitations, they can attempt to revise textbooks to provide better OTL to teachers and students.

References

Alkhrausi, H. (2012). Generalizability theory: An analysis of variance approach to measurement problems in educational assessment. *Journal of Studies in Education, 2*(1), 184-196.

Battista, M. (2004). Applying Cognition-Based Assessment to Elementary School Students' Development of Understanding of Area and Volume Measurement. *Mathematical Thinking and Learning, 6*(2), 185-204. doi:10.1207/s15327833mtl0602_6

Battista, M., & Clements, D. (1996). Students' Understanding of Three-Dimensional Rectangular Arrays of Cubes. *Journal for Research in Mathematics Education, 27*(3), 258-292. doi:10.2307/749365

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31*(3), 235-268. doi:10.1007/bf00376322

Cady, J. A., Hodges, T. E., & Collins, R. L. (2015). A Comparison of Textbooks' Presentation of Fractions. *School Science and Mathematics, 115*(3), 105-116. doi:10.1111/ssm.12108

Cai, J. (2005). U.S. and Chinese Teachers' Constructing, Knowing, and Evaluating Representations to Teach Mathematics. *Mathematical Thinking and Learning, 7*(2), 135-169. doi:10.1207/s15327833mtl0702_3

Cai, J., & Nie, B. (2007). Problem solving in Chinese mathematics education: research and practice. *ZDM, 39*(5), 459-473. doi:10.1007/s11858-007-0042-3

Charalambous, C. Y., Delaney, S., Hsu, H.-Y., & Mesa, V. (2010). A Comparative Analysis of the Addition and Subtraction of Fractions in Textbooks from Three Countries. *Mathematical Thinking and Learning, 12*(2), 117-151. doi:10.1080/10986060903460070

Chingos, M. M., & Whitehurst, G. J. (2012). *Choosing blindly: Instructional materials, teacher effectiveness, and the Common Core*. Retrieved from Washington, DC:

Curry, M., Mitchelmore, M., & Outhred, L. (2006). *Development of children's understanding of length, area, and volume measurement principles.* Paper presented at the the Thirtieth Annual Meeting of the International Group for the Psychology of Mathematics Education, Prague, Czech Republic.

Ding, M. (2016). Opportunities to Learn: Inverse Relations in U.S. and Chinese Textbooks. *Mathematical Thinking and Learning, 18*(1), 45-68. doi:10.1080/10986065.2016.1107819

Floden, R. (2002). The measurement of opportunity to learn. In A. Porter & A. Gamoran (Eds.), *Methodological advances in cross-national surveys of educational assessment* (pp. 231-266). Washington, DC: National Academy Press.

Ha, S., Pang, j., & Ju, M.-k. (2010). research Trends in Elementary Mathematics Education: Focused on the Papers Published in Domestic Journals Duing the Recent 5 years. *Journal of Korean Society of Mathematics Education Series A: The Mathematical Education, 49*(1), 67-83.

Hiebert, J., & Lefvre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum.

Hong, D. S., & Choi, K. M. (2018). A comparative analysis of linear functions in Korean and American standards-based secondary textbooks. *International Journal of Mathematical Education in Science and Technology, 49*(7), 1025-1051. doi:10.1080/0020739X.2018.1440327

Hong, D. S., Choi, K. M., Runnalls, C., & Hwang, J. (2018). Do textbooks address known learning challenges in area measurement? A comparative analysis. *Mathematics Education Research Journal, 30*(3), 325-354. doi:10.1007/s13394-018-0238-6

Hong, D. S., Choi, K. M., Runnalls, C., & Hwang, J. (2019). How well aligned are common core textbooks to students' development in area measurement? *School Science and Mathematics, 119*(5), 240-254. doi:10.1111/ssm.12336

Kim, H. J. (2013). *A Survey on the Reasoning Ability of the Fourth and Fifth Graders in the Context of Volume Comparison.* (Masters), Graduate School of Education Korea National University of Education Chung-Buk, KOREA,

Lee, E. H. (2006). *An Analysis on the Measurement Sense and Measurement Capability of the Sixth Graders.* (Masters), Graduate School of Education Korea National University of Education Chung-Buk, KOREA,

Lee, J. (2010). Children's strategies for measurement estimation of rectangular covering tasks. *Journal of the Korean Society of Mathematical Education series A, 49*(3), 375-487.

Lee, K., & Smith, J. P. (2011). What is different across an ocean? How Singapore and US elementary mathematics curricula introduce and develop length measurement. *ZDM, 43*(5), 681. doi:10.1007/s11858-011-0339-0

Lehrer, R. (2003). Developing understanding of measurement. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 179-192). Reston, VA: National Council of Teachers of Mathematics.

Liu, X. (2009). Competence and Opportunity to Learn. In X. Liu (Ed.), *Linking Competence to Opportunities to Learn: Models of Competence and Data Mining* (pp. 5-11). Dordrecht: Springer Netherlands.

Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *The TIMSS 2011 International Results in Mathematics*: Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 International Results in Mathematics*. Retrieved from Boston College: http://timssandpirls.bc.edu/timss2015/international-results/

Murphy, C. (2012). The role of subject knowledge in primary prospective teachers' approaches to teaching the topic of area. *Journal of Mathematics Teacher Education, 15*(3), 187-206. doi:10.1007/s10857-011-9194-8

National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to actions : ensuring mathematical success for all*: Reston, VA : NCTM, National Council of Teachers of Mathematics, [2014] ©2014.

Neidorf, T. S., Binkley, M., Gattis, K., & Nohara, D. (2006). *Comparing Mathematics Content in the National Assessment of Educational Progress (NAEP), Trends in International Mathematics and Science Study (TIMSS), and Program*

for International Student Assessment (PISA) 2003 Assessments. Retrieved from Washington, D.C.:

Nie, B., Cai, J., & Moyer, J. C. J. Z. (2009). How a standards-based mathematics curriculum differs from a traditional curriculum: with a focus on intended treatments of the ideas of variable. *41*(6), 777-792. doi:10.1007/s11858-009-0197-1

Otten, S., Gilbertson, N. J., Males, L. M., & Clark, D. L. (2014). The Mathematical Nature of Reasoning-and-Proving Opportunities in Geometry Textbooks. *Mathematical Thinking and Learning, 16*(1), 51-79. doi:10.1080/10986065.2014.857802

Outhred, L., & Mitchelmore, M. (2000). Young Children's Intuitive Understanding of Rectangular Area Measurement. *Journal for Research in Mathematics Education, 31*(2), 144-167. doi:10.2307/749749

Pang, J. (2014). Current Elementary Mathematics Textbooks. In J. Kim, I. Han, & J. Lee (Eds.), *Mathematics Education in Korea - Vol. 1 Curricular and Teaching and Learning Practices* (pp. 43-61): World Scientific Publishing Company.

Polikoff, M. S. (2015). How Well Aligned Are Textbooks to the Common Core Standards in Mathematics? *American Educational Research Journal, 52*(6), 1185-1211. doi:10.3102/0002831215584435

Remillard, J. T. (2005). Examining Key Concepts in Research on Teachers' Use of Mathematics Curricula. *Review of Educational Research, 75*(2), 211-246. doi:10.3102/00346543075002211

Remillard, J. T., Harris, B., & Agodini, R. (2014). The influence of curriculum material design on opportunities for student learning. *ZDM, 46*(5), 735-749. doi:10.1007/s11858-014-0585-z

Remillard, J. T., & Heck, D. J. (2014). Conceptualizing the curriculum enactment process in mathematics education. *ZDM, 46*(5), 705-718. doi:10.1007/s11858-014-0600-4

Roth McDuffie, A., Choppin, J., Drake, C., & Davis, J. (2018). Middle school mathematics teachers' orientations and noticing of features of mathematics curriculum materials. *International Journal of Educational Research*. doi:https://doi.org/10.1016/j.ijer.2018.09.019

Roth McDuffie, A., Choppin, J., Drake, C., Davis, J. D., & Brown, J. (2017). Middle school teachers' differing perceptions and use of curriculum materials and the common core. *Journal of Mathematics Teacher Education*. doi:10.1007/s10857-017-9368-0

Sahm, C. (2015). *Curriculum counts: NYC public schools and the Common Core. Civic Report*. Retrieved from New York, NY: https://www.manhattan-institute.org/html/curriculum-counts-nyc-public-schools-and-common-core-6360.html.

Sarama, J., & Clements, D. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York: Routledge.

Smith, J. P., Males, L. M., Dietiker, L. C., Lee, K., & Mosier, A. (2013). Curricular Treatments of Length Measurement in the United States: Do They Address Known Learning Challenges? *Cognition and Instruction, 31*(4), 388-433. doi:10.1080/07370008.2013.828728

Smith, J. P., Males, L. M., & Gonulates, F. (2016). Conceptual Limitations in Curricular Presentations of Area Measurement: One Nation's Challenges. *Mathematical Thinking and Learning, 18*(4), 239-270. doi:10.1080/10986065.2016.1219930

Son, J.-W., & Hu, Q. (2016). The initial treatment of the concept of function in the selected secondary school mathematics textbooks in the US and China. *International Journal of Mathematical Education in Science and Technology, 47*(4), 505-530. doi:10.1080/0020739X.2015.1088084

Son, J.-W., & Kim, O.-K. (2015). Teachers' selection and enactment of mathematical problems from textbooks. *Mathematics Education Research Journal, 27*(4), 491-518. doi:10.1007/s13394-015-0148-9

Son, J.-W., & Senk, S. L. (2010). How reform curricula in the USA and Korea present multiplication and division of fractions. *Educational Studies in Mathematics, 74*(2), 117-142. doi:10.1007/s10649-010-9229-6

Stein, M. K., Remillard, J. T., & Smith, M. S. (2007). How curriculum influences student learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319 – 369). Greenwich, CT: Information Age Publishing.

Vasilyeva, M., Ganley, C. M., Casey, B. M., Dulaney, A., Tillinger, M., & Anderson, K. (2013). How Children Determine the Size of 3D Structures: Investigating Factors Influencing Strategy Choice. *Cognition and Instruction, 31*(1), 29-61. doi:10.1080/07370008.2012.742086

Wang, Y., Barmby, P., & Bolden, D. (2017). Understanding Linear Function: a Comparison of Selected Textbooks from England and Shanghai. *International Journal of Science and Mathematics Education, 15*(1), 131-153. doi:10.1007/s10763-015-9674-x

Zacharos, K. (2006). Prevailing educational practices for area measurement and students' failure in measuring areas. *The Journal of Mathematical Behavior, 25*(3), 224-239. doi:https://doi.org/10.1016/j.jmathb.2006.09.003

Zhu, Y., & Fan, L. (2006). Focus on the Representation of Problem Types in Intended Curriculum: A Comparison of Selected Mathematics Textbooks from Mainland China and the United States. *International Journal of Science Mathematics Education, 4*(4), 609-626. doi:10.1007/s10763-006-9036-9

Appendix. Textbooks Analyzed

Go math! Common Core edition, Grade 2 (2015 a) (student edition e-book). Orlando, FL: Houghton Mifflin Harcourt

Go math! Common Core edition, Grade 3 (2015 b) (student edition e-book). Orlando, FL: Houghton Mifflin Harcourt

Go math! Common Core edition, Grade 4 (2015 c) (student edition e-book). Orlando, FL: Houghton Mifflin Harcourt

Go math! Common Core edition, Grade 5 (2015 d) (student edition e-book). Orlando, FL: Houghton Mifflin Harcourt

MyMath, Grade 3. (2014 a). New York, NY: Macmillan/McGraw-Hill.

MyMath, Grade 4. (2014 b). New York, NY: Macmillan/McGraw-Hill.

MyMath, Grade 5. (2014 c). New York, NY: Macmillan/McGraw-Hill.

The Ministry of Education in Korea (2014a) Mathematics 1. Seoul, Korea.

The Ministry of Education in Korea (2014b) Mathematics 3. Seoul, Korea.

The Ministry of Education in Korea (2015) Mathematics 6. Seoul, Korea.